

Factoring Polynomials

This section will cover the following topics

Factoring the Greatest Common Factor

Factoring Trinomials by Trial and Error

Solving Equations by Factoring

Factoring the Greatest Common Factor

The most basic type of factoring for polynomials is to factor out the **Greatest Common Factor (GCF)**. The goal of factoring is to undo multiplication. Let's take a look at what multiplying a single term into a polynomial looks like, and then we will work backwards.

Example of Multiplication of a Polynomial by a Single Term

$3x^2(2x^3 - 3x^2 + 5x - 4)$	We must first distribute the $3x^2$ to each term inside the parentheses, and then multiply term by term.
$3x^2 \cdot 2x^3 - 3x^2 \cdot 3x^2 + 3x^2 \cdot 5x - 3x^2 \cdot 4$	
$6x^5 - 9x^4 + 15x^3 - 12x^2$	

Working backwards, let's start with the polynomial $6x^5 - 9x^4 + 15x^3 - 12x^2$. When factoring the GCF deal with the numbers and each variable separately to determine the overall GCF.

Finding the GCF of $6x^5 - 9x^4 + 15x^3 - 12x^2$

GCF of the Coefficients (dealing with the numbers)	6, 9, 15, and 12 are the coefficients All of these numbers are divisible by 1 and 3 only. Always take the highest number, which in this case is 3
GCF of the Variable (dealing with the <i>x</i> - <i>variable</i>)	These include $x^5, x^4, x^3,$ and x^2 To find the GCF of variables, take the variable raised to the lowest exponent. In this case, that is x^2
The Overall GCF	Putting the GCF of the numbers and variables together, we get $GCF = 3x^2$
Factor the GCF Start by factoring $3x^2$ from each term. Then factor $3x^2$ outside parentheses and the remaining terms inside	$6x^5 - 9x^4 + 15x^3 - 12x^2$ $3x^2 \cdot 2x^3 - 3x^2 \cdot 3x^2 + 3x^2 \cdot 5x - 3x^2 \cdot 4$ $3x^2(2x^3 - 3x^2 + 5x - 4)$

Factoring Trinomials by Trial and Error

Once again, we will start with the idea that factoring will undo multiplication. For trinomials (polynomials with three terms), this means we will be undoing FOIL-ing (see the review on Polynomials for details).

Example of FOIL-ing

First Term · First Term	$(x + 3)(x - 7) = x \cdot x$
Outside Term · Outside Term	$(x + 3)(x - 7) = x^2 - 7 \cdot x$
Inside Term · Inside Term	$(x + 3)(x - 7) = x^2 - 7x + 3 \cdot x$
Last Term · Last Term	$(x + 3)(x - 7) = x^2 - 7x + 3x - 21$
Finally, combine any like terms	$(x + 3)(x - 7) = x^2 - 4x - 21$

To work backwards, we will start by considering the possible ways to factor the first term x^2 and the last term -21 . We will then write all possible factorizations based on those

Example: Factor $x^2 - 4x - 21$

Possible Factors of First Term	Possible Factors of Last Term	Possible Factorization	Check by FOIL-ing
$x^2 = x \cdot x$	$-21 = -3 \cdot 7$	$(x - 3)(x + 7)$	$x^2 + 4x - 21$
$x^2 = x \cdot x$	$-21 = 3 \cdot -7$	$(x + 3)(x - 7)$	$x^2 - 4x - 21^*$
$x^2 = x \cdot x$	$-21 = -1 \cdot 21$	$(x - 1)(x + 21)$	$x^2 + 20x - 21$
$x^2 = x \cdot x$	$-21 = 1 \cdot -21$	$(x + 1)(x - 21)$	$x^2 - 20x - 21$

*Note that we could have stopped at the second row because we found the factorization.

Example 2: Factor $2x^2 - 5x + 3$

Possible Factors of First Term	Possible Factors of Last Term	Possible Factorization	Check by FOIL-ing
$2x^2 = 2x \cdot x$	$3 = 1 \cdot 3$	$(2x + 1)(x + 3)$	$2x^2 + 7x + 3$
		$(2x + 3)(x + 1)$	$2x^2 + 5x + 3$
$2x^2 = 2x \cdot x$	$3 = -1 \cdot -3$	$(2x - 1)(x - 3)$	$2x^2 - 7x + 3$
		$(2x - 3)(x - 1)$	$2x^2 - 5x + 3$



A Quick Tip

Factoring can get complicated very quickly, and so can factoring techniques. On the placement exam, keep it simple. These represent the difficulty level you will find on the exam.

Solving Equations by Factoring

A very important point about solving equations by factoring is that one side of the equation must be equal to zero. Once you have that, solving equations by factoring is easy; simply factor and then set each factor equal to zero.

Example 1: $2x^2 - 6x = 0$

Factor out the GCF	$2x(x - 3) = 0$	
Set each factor equal to zero	$2x = 0$	$x - 3 = 0$
Solve each equation	$x = 0$	$x = 3$

Example 2: $3x^2 - 5x = 2$

Write equation with = 0	$3x^2 - 5x - 2 = 0$	
Factor	$(3x + 1)(x - 2) = 0$	
Set each factor equal to zero	$3x + 1 = 0$	$x - 2 = 0$
Solve each equation	$x = -\frac{1}{3}$	$x = 2$

Practice Problems

Factor the following expressions, or factoring to solve the following equations

1. $6x^5 + 9x^4 - 24x^3 + 18x^2$	2. $x^2 - 4x - 32$	3. $3x^2 + 14x - 5$
4. $3x^2 - 4x = 0$	5. $x^2 - 3x - 28 = 0$	6. $2x^2 - 5x = 7$

Answers

1. $3x^2(2x^3 + 3x^2 - 8x + 6)$	2. $(x - 8)(x + 4)$	3. $(3x - 1)(x + 5)$
4. $x = 0, \frac{4}{3}$	5. $x = 7, -4$	6. $x = -1, \frac{7}{2}$



Additional Help

You can also search YouTube.com for “factoring the GCF”, “factoring trinomials”, or “solving quadratic equations by factoring”