

Fractions

This section will cover the following topics

Reducing to Lowest Terms

Converting between Mixed Numbers and Improper Fractions

Multiplying and Dividing

Adding and Subtracting

Reducing

Fractions have the ability to look different without changing value. A common example that is given is slicing a pizza. Suppose someone was feeling very hungry and wanted a very large slice of pizza so they cut the pizza into two slices by cutting right down the middle. Eating only one of those very large slices would mean eating exactly $\frac{1}{2}$ of the pizza. But if the pizza were bought from a shop where it was already sliced into eight pieces, that person could easily still eat $\frac{1}{2}$ of the pizza by eating 4 out of 8 slices. In fractions, that means $\frac{4}{8} = \frac{1}{2}$. Even though these fractions look quite different, they still represent the same value. Here are some examples to illustrate how fractions are reduced, and what it means to be reduced to **lowest terms**. The key is to look for common divisors.

Example 1

$$\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$$

In looking at the fraction $\frac{6}{9}$, we are looking for a number which will divide both 6 and 9, which is **3**. 3 is called a common divisor. $\frac{2}{3}$ is in lowest terms because 2 and 3 do not share a common divisor, so we are done.

Example 2

$$\frac{24}{30} = \frac{24 \div 6}{30 \div 6} = \frac{4}{5}$$

$\frac{24}{30}$ is a bit more challenging because there are many numbers which divide both 24 and 30. They are 2, 3, and 6. We will use **6** to do the reduction because it is the greatest; called the greatest common divisor.

Example 3

$$\begin{aligned} \frac{216}{240} &= \frac{216 \div 2}{240 \div 2} = \frac{108 \div 2}{120 \div 2} \\ &= \frac{54 \div 2}{60 \div 2} = \frac{27 \div 3}{30 \div 3} = \frac{9}{10} \end{aligned}$$

$\frac{216}{240}$ looks nearly impossible, but will turn out to be much easier than it looks if we take it step-by-step. That is, let's start with dividing both 216 and 240 by **2** since they are both even numbers. From there we will continue by dividing by **2**, **2**, and finally **3**.



A Quick Tip

Whenever doing math, either in a class or on the placement exam, fractions are always reduced.

Converting between Mixed Numbers and Improper Fractions

Let's start by giving an example of a mixed number and an improper fraction. $3\frac{1}{2}$ is a **mixed number** because it mixes a whole number (the 3) with a fraction (the $\frac{1}{2}$). The fraction $\frac{7}{2}$ is an **improper fraction** because the numerator is larger than the denominator. We can convert between these two forms in the following ways.

Convert a Mixed Number to an Improper Fraction: $3\frac{1}{2}$ to $\frac{7}{2}$

First multiply the whole 3 by the denominator 2 .	$3\frac{1}{2} : 3 \times 2 = 6$
Then take the resulting 6 and add the numerator of 1 .	$3\frac{1}{2} : 6 + 1 = 7$
The 7 become the new numerator and the 2 remains the denominator.	$\frac{7}{2}$

Convert an Improper Fraction to a Mixed Number: $\frac{7}{2}$ to $3\frac{1}{2}$

First we must determine how many times 2 goes into 7 without going over 7. The answer (highlighted in yellow) is 3 times with 1 left over.	$\frac{7}{2} :$ <table style="display: inline-table; vertical-align: middle;"> <tr><td>2 × 1 = 2</td></tr> <tr><td>2 × 2 = 4</td></tr> <tr><td>2 × 3 = 6</td></tr> <tr><td>2 × 4 = 8</td></tr> </table>	2 × 1 = 2	2 × 2 = 4	2 × 3 = 6	2 × 4 = 8
	2 × 1 = 2				
2 × 2 = 4					
2 × 3 = 6					
2 × 4 = 8					
	$\frac{7}{2} = 3\frac{1}{2} :$				

Multiplying and Dividing

Multiplication and Division are relatively straightforward. For multiplication remember to multiply across (numerator × numerator and denominator × denominator) and then reduce. For division of fractions, simply change division to multiplication. To change division into multiplication, remember Copy – Dot – Flip. That is, Copy the first fraction – change division (÷) to multiplication (·) – and flip the second fraction ($\frac{a}{b} \rightarrow \frac{b}{a}$).

Multiplication Example 1: $\frac{4}{5} \cdot \frac{3}{8}$		Multiplication Example 2: $\frac{10}{6} \cdot \frac{9}{12}$	
$\frac{4}{5} \cdot \frac{3}{8} = \frac{12}{40}$	Multiply Across	$\frac{10}{6} \cdot \frac{9}{12} = \frac{90}{72}$	Multiply Across
$\frac{12 \div 4}{40 \div 4} = \frac{3}{10}$	Reduce to Lowest Terms	$\frac{90 \div 18}{72 \div 18} = \frac{5}{4}$	Reduce to Lowest Terms

Division Example 1: $\frac{5}{6} \div \frac{10}{7}$		Division Example 2: $\frac{8}{9} \div \frac{6}{5}$	
$\frac{5}{6} \div \frac{10}{7} = \frac{5}{6} \cdot \frac{7}{10}$	Copy – Dot – Flip	$\frac{8}{9} \div \frac{6}{5} = \frac{8}{9} \cdot \frac{5}{6}$	Copy – Dot – Flip
$\frac{5}{6} \cdot \frac{7}{10} = \frac{50}{42}$	Multiply Across	$\frac{8}{9} \cdot \frac{5}{6} = \frac{40}{54}$	Multiply Across
$\frac{50 \div 2}{42 \div 2} = \frac{25}{21}$	Reduce to Lowest Terms	$\frac{40 \div 2}{54 \div 2} = \frac{20}{27}$	Reduce to Lowest Terms

Adding and Subtracting

While multiplying and dividing fractions are straightforward, adding and subtracting fractions are not – UNLESS – you have a common denominator between the two fractions. So that's the trick – get a common denominator before adding/subtracting.

Addition Example: $\frac{2}{3} + \frac{1}{4}$		Subtraction: $\frac{5}{6} - \frac{2}{9}$	
$\frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12}$	The common denominator is 12 . Multiply first fraction by 4 's and the second by 3 's	$\frac{5 \cdot 3}{6 \cdot 3} = \frac{15}{18}$	The common denominator is 18 . Multiply first fraction by 3 's and the second by 2 's
$\frac{1 \cdot 3}{4 \cdot 3} = \frac{3}{12}$		$\frac{2 \cdot 2}{9 \cdot 2} = \frac{4}{18}$	
$\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$	Add the numerators and reduce if necessary	$\frac{15}{18} - \frac{4}{18} = \frac{11}{18}$	Subtract the numerators and reduce if necessary



A Quick Tip

It is easy to confuse the techniques between multiplying/dividing fractions and adding/subtracting fractions, and it often is related to when a common denominator is needed. Remember that a common denominator is needed only for addition and subtraction of fractions.

Practice Problems

Reduce the following fractions to lowest terms

1. $\frac{12}{15}$

2. $\frac{32}{40}$

3. $\frac{12}{16}$

Convert the following mixed numbers to improper fractions, and improper fractions to mixed numbers.

4. $2\frac{1}{3}$

5. $5\frac{3}{7}$

6. $\frac{22}{3}$

7. $\frac{19}{8}$

Perform the following multiplication/division problems with fractions.

$$8. \frac{2}{5} \cdot \frac{10}{3}$$

$$9. \frac{6}{9} \cdot \frac{3}{8}$$

$$10. \frac{3}{4} \div \frac{1}{2}$$

$$11. \frac{2}{7} \div \frac{5}{14}$$

Perform the following addition/subtraction problems with fractions.

$$12. \frac{1}{4} + \frac{3}{5}$$

$$13. \frac{4}{9} + \frac{2}{6}$$

$$14. \frac{5}{6} - \frac{3}{4}$$

$$15. \frac{4}{5} - \frac{2}{3}$$

Answers

Reduce the following fractions to lowest terms

$$1. \frac{4}{5}$$

$$2. \frac{4}{5}$$

$$3. \frac{3}{4}$$

$$4. \frac{7}{3}$$

$$5. \frac{38}{7}$$

$$6. 7\frac{1}{3}$$

$$7. 2\frac{3}{8}$$

$$8. \frac{4}{3}$$

$$9. \frac{1}{4}$$

$$10. \frac{3}{2}$$

$$11. \frac{4}{5}$$

$$12. \frac{17}{20}$$

$$13. \frac{7}{9}$$

$$14. \frac{1}{12}$$

$$15. \frac{2}{15}$$



Additional Help

You can also search YouTube.com for “reducing fractions”, “converting fractions”, or “adding/subtracting/multiplying/dividing fractions”