Polynomials

This section will cover the following topics

Definitions; "Polynomial", "Like Terms" and "Combine Like Terms"
Adding and Subtracting Polynomials
Multiplying Polynomials
Dividing a Polynomial by a Monomial

Definitions; "Polynomial", "Like Terms" and "Combine Like Terms"

A simple way to think of a **Polynomial** is that it is an expression that combines numbers and variables through addition, subtraction and multiplication. It is important to note that division is missing from this list. Additionally, this also implies that the exponent of each variable must be a positive counting number ($1, 2, 3, 4, \ldots$). Here are some examples to illustrate this.

Expressions	Polynomial (Yes or No?)
$x^3 - x^2 + 2x + 7$	**Yes –** only addition/subtract/multiplication are used, and the exponents are positive counting numbers
$2x^{-5} + \frac{7}{y^2} - \sqrt{x}$	**No –** this expression includes the negative exponent "-5", division by y, and a square root – all of which are not allowed for polynomials
$2x^5 + 7y^2$	**Yes –** even though there are multiple variables (x and y) this is still a polynomial

Like Terms are parts of an expression that share the same variable (or variables) and each of those variables has the same exponent.

Terms	Like Terms (Yes or No?)
$2x^2y^3z \text{ and } 7x^2y^3z$	**Yes –** both terms share the same variables with the same exponents; $x^2, y^3, \text{and } z$
$2x^2y^3z \text{ and } 7x^2y^3$	**No –** the first term contains the variable z, however the second terms does not
$2x^2y^3z \text{ and } 7x^4y^3z$	**No –** although the variables are shared by both terms, the exponent of the x variable is "2" in the first term and "4" in the second term

Now that we know what Like Terms are, we can define the phrase, "**Combine Like Terms**". Think apples and oranges; Adding 2 apples to 7 apples to get 9 apples is combining like terms, however we cannot add 2 apples with 7 oranges because they are not "like". Just like two sets of apples can be added (or subtracted), so can like terms.

Examples

$x^2 + x^2 = 2x^2$	$10x^2\text{y} + 5x^2y = 15x^2y$	Note that combining like terms is just adding/subtracting the numbers; e.g. $8 - 2 = 6$
$4x^2 - 9x^2 = -5x^2$	$8x^2y^3z - 2x^2y^3z = 6x^2y^3z$	

Adding and Subtracting Polynomials

Adding and Subtracting Polynomials is really just combining like terms with one exception; that exception will be highlighted in examples 2 and 3. Let's look at some examples.

More Examples

Example 1

$3x^2 + 7x - 5 + 5x^3 - 4x^2 + 7x + 11$	First look for terms with the same variables AND exponents (done by color), and then add/subtract as needed. If a term is by itself, like $5x^3$, there is no need to do anything
$3x^2 + 7x - 5 + 5x^3 - 4x^2 + 7x + 11$	
$5x^3 - x^2 + 14x + 6$	

Example 2

$-(2x^2 - 5x + 2) + 4(6x^2 - 9x + 1)$	In this case, there are parentheses that need to be removed before collecting like terms. This is done by distributing the $-$ and the 4, which means multiplying every term inside the first set of parentheses by $-$ and every term in the second set of parentheses by 4. Once finished, collect like terms.
$-2x^2 + 5x - 2 + 24x^2 - 36x + 4$	
$22x^2 - 31x + 2$	

A Quick Recap

When adding/subtracting polynomials, remember apples and oranges. If terms have the same variables with the same exponents, they are like terms and can be added/subtracted.

Multiplying Polynomials

Let's start with a quick lesson on multiplying two single term expressions together. We will use the following rule; $x^a \cdot x^b = x^{a+b}$. That is when two expressions with the same base are multiplied (x is the base, a and b are the exponents) we can add the exponents together. Let's look at a few examples.

Examples

$2x^2 \cdot 5x^3 = 10x^5$	$12x^2y^5 \cdot 3x^7y = 36x^9y^6$	Multiply the numbers as usual. Then match up the variables (x with x, y with y, etc.) and add their exponents.
$8x^3y^3z^5 \cdot 2x^2y^7z^2 = 16x^5y^{10}z^7$		

Next, let's take a look at multiplying a single term by a polynomial with multiple terms.

Example

$3x^2(2x^3 - 3x^2 + 5x - 4)$
$3x^2 \cdot 2x^3 - 3x^2 \cdot 3x^2 + 3x^2 \cdot 5x - 3x^2 \cdot 4$

$6x^5 - 9x^4 + 15x^3 - 12x^2$	We must first distribute the $3x^2$ to each term inside the parentheses, and then multiply as we did in the last example.

To finish the multiplication of polynomials, we will multiply two binomials together using the technique called FOIL. FOIL gives the order in which the terms of each binomial should be multiplied; First – Outside – Inside – Last

Example

First Term · First Term	$(\boldsymbol{x} + 3)(\boldsymbol{x} - 7) = \boldsymbol{x} \cdot \boldsymbol{x}$
Outside Term · Outside Term	$(\boldsymbol{x} + 3)(x - \mathbf{7}) = x^2 - \mathbf{7} \cdot \boldsymbol{x}$
Inside Term · Inside Term	$(x + \mathbf{3})(\boldsymbol{x} - 7) = x^2 - 7x + \mathbf{3} \cdot \boldsymbol{x}$
Last Term · Last Term	$(x + \mathbf{3})(x - \mathbf{7}) = x^2 - 7x + 3x - \mathbf{21}$
Finally, combine any like terms	$(x + 3)(x - 7) = x^2 - 4x - 21$

Dividing a Polynomial by a Monomial

Dividing a Polynomial by a Monomial means dividing a polynomial by a single term. Here are a couple of examples of what that looks like.

Example 1

$\frac{10x^7}{2x^2}$	This is an examles of dividing a single term polynomial by a single term. It is best to work with numbers and variables separately.
$\frac{\mathbf{10}x^7}{\mathbf{2}x^2} = \frac{\mathbf{5}x^7}{x^2}$	**10** ÷ **2** = **5**
$\frac{5x^{\mathbf{7}}}{x^{\mathbf{2}}} = 5x^{\mathbf{5-2}} = 5x^3$	Then subtract the exponent of x in the denominator from the exponent in the numerator.

Example 2

$\frac{10x^5 - 4x^4 + 7x^3 + 2x^2}{2x^2}$	This is an example of dividing a polynomial with multiple terms by a single term. We will again work with numbers and variables separately.
$\frac{10x^5}{2x^2} - \frac{4x^4}{2x^2} + \frac{7x^3}{2x^2} + \frac{2x^2}{2x^2}$	Before we do that, we must split the polynomial in the numerator.
$5x^3 - 2x^2 + \frac{7}{2}x + 1$	Once that is done, work term by term using the same techniques as was done in example 1.

A Quick Recap

When multiplying/dividing polynomials, remember to work with numbers and variables separately. When working with variables, exponents are added for multiplication and subtracted for division

Practice Problems

Perform the following polynomial arithmetic

1. $2(3x^2 - 8x + 9) - 3(6x^2 - 4x + 1)$	2. $7(5x^3 - x^2 + 4x) - (2x^2 - 3x + 4)$
3. $2x^2(6x^2 - 4x + 1)$	4. $(2x + 7)(3x - 1)$
5. $\frac{9x^4 - x^3 + 3x^2}{3x^2}$	

Answers

1. $-12x^2 - 4x + 15$	2. $35x^3 - 9x^2 + 31x - 4$
3. $12x^4 - 8x^3 + 2x^2$	4. $6x^2 + 19x - 7$
5. $3x^2 - \frac{1}{3}x + 1$	

Additional Help

You can also search YouTube.com for “adding polynomials”, “multiplying polynomials”, or “dividing polynomials”