

Solving Linear Equations

This section will cover the following topics

What is a Linear Equation?

Solving One-Step Linear Equations

Solving Two-Step Linear Equations

Solving Linear Equations That Include Parentheses

What is a Linear Equation?

A linear equation is an equation in which the variable (or variables) have an exponent of 1, and in which the variables do not appear in the denominator. Here are some examples of equations that are both linear and non-linear

Equation	Linear (Yes or No?)
$x + 7 = 5$	Yes – the x does not appear to have an exponent, but in fact there is an implied exponent of 1. That is, when we write x , we mean x^1 .
$5x + 7y = 10$	Yes – Even though there are multiple variables, they each have an implied exponent of 1.
$x^2 + 7 = 5$	No – In this case, the exponent of the x is 2. Thus, the equation is not a linear equation.
$\frac{1}{x} + 7x = 5$	No – Although the x variable has no visible exponent, it is in the denominator of the first term. Thus, the equation is not a linear.

Solving One-Step Linear Equations

A key thing to keep in mind is that solving an equation means to isolate the variable on one side of the equals sign. That is, the end result should look like, " $x = \underline{\hspace{2cm}}$ ". A One-Step Linear Equation is a linear equation that is a single operation away from being solved; either through addition, subtraction, multiplication, or division. Here are some examples

Example 1

$x + 7 = 5$	In this example, the only step that needs to be done is to eliminate the +7 from the left hand side of the equation. This is done by the " -7 " – remember that what is done to one of the equation must be done to both.
$x + 7 - 7 = 5 - 7$	
$x = -2$	

Example 2

$2x = 10$	In this example, the only step that needs to be done is to eliminate the 2 in front of the x . This is done by dividing both sides of the equation by 2.
$\frac{2x}{2} = \frac{10}{2}$	
$x = 5$	

Solving Two-Step Linear Equations

Solving Two-Step Linear Equations puts together the pieces in the above examples to solve a single problem. In other words, we will (1) add/subtract and then (2) multiply/divide. It will be done in that order, too.

Example 1

$2x + 7 = 5$	<p>In this example, the two steps that will need to be done are to divide by 2 and subtract 7 from both sides.</p> <p>We must do the “$- 7$” first.</p> <p>And will finish by dividing both sides by 2.</p>
$2x + 7 - 7 = 5 - 7$	
$2x = -2$	
$\frac{2x}{2} = \frac{-2}{2}$	
$x = -1$	

Example 2

$\frac{x}{2} - 3 = 5$	<p>In this example, the two steps that will need to be done are to multiply by 2 and add 3 to both sides.</p> <p>We must do the “$+ 3$” first.</p> <p>And will finish by multiplying both sides by 2.</p>
$\frac{x}{2} - 3 + 3 = 5 + 3$	
$\frac{x}{2} = 8$	
$2 \cdot \frac{x}{2} = 2 \cdot 8$	
$x = 16$	

Solving Linear Equations That Include Parentheses

To solve linear equations that involve parentheses, the first thing we must do is eliminate the parentheses on each side of the equation and then combine like terms. At that point, all we need to do is apply the same techniques we have already been doing.

Example 1

$5(x + 2) - x = 14$	<p>First, distribute the 5 over the parentheses</p> <p>Then, combine the terms $5x$ and $-x$.</p> <p>To finish, use the techniques from above; subtract 10 from each side followed by dividing both sides by 4.</p>
$5x + 10 - x = 14$	
$4x + 10 = 14$	
$4x = 4$	
$x = 1$	

Example 2

$3 + 7(x - 1) = 2$	<p>First, distribute the 7 over the parentheses</p> <p>Then, combine the terms 3 and -7.</p> <p>To finish, use the techniques from above; add 4 to each side followed by dividing both sides by 7.</p>
$3 + 7x - 7 = 2$	
$-4 + 7x = 2$	
$7x = 6$	
$x = \frac{6}{7}$	



A Quick Tip

If you get an equation to solve on the placement exam, either solve it directly or check each possible answer using substitution as an alternative strategy. Choose the strategy that gives you the best chance to succeed!

Practice Problems

Solve the following Linear Equations

1. $x + 13 = 4$	2. $x - 7 = 15$	3. $5x = 15$
4. $3x = -14$	5. $2x - 7 = 3$	6. $-5x + 2 = -13$
7. $3(2x + 4) - 5 = 9$	8. $6 - 4(x + 2) + 3x = 1$	

Answers

1. $x = -9$	2. $x = 22$	3. $x = 3$	4. $x = -\frac{14}{3}$
5. $x = 5$	6. $x = 3$	7. $x = \frac{1}{3}$	8. $x = -3$



Additional Help

You can also search YouTube.com for “solving linear equations”